

Validating Risk-Neutral Scenarios: The Martingale Report

by William Pauling, CFA

Introduction

The Martingale test is used in the validation of risk-neutral scenarios. The test checks that, for a set of cash flows, the discounted value is equal to 1.0. For a set of cash flows, the Initial Price = $E(\sum \text{Cash Flow}_T * \text{Discount}_T)$. If we consider a strategy that buys and holds \$1 of a specific investment for N years, then our $\text{Cash Flow}_N = 1 + \text{Cumulative Return}_N$ for that investment, which leads to $1 = E((1 + \text{Cumulative Return}_N) * \text{Discount}_N)$. Thus, the average result of a Martingale test should be around 1.0. For this reason, the Martingale test is sometimes referred to as a “1 = 1” test.

Each quarter, Conning publishes a risk-neutral validation package to accompany the updated risk-neutral calibrations. The report includes a Martingale test for each of the standard asset classes.

Interpreting the Results

As mentioned previously, we expect to see the average discounted value around 1.0. In addition to the average, the graph also displays the upper and lower 5th percentiles for a 90% confidence interval. It is important to use a confidence interval that considers sampling error. Our confidence interval is calculated from the Standard Error (SE) of the test measure, which is:

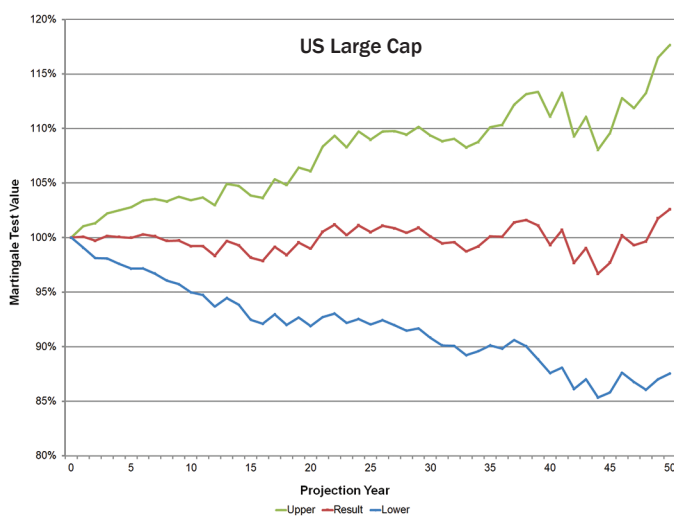
$$SE = \frac{\sigma}{\sqrt{N}}$$

The upper and lower 5% is +/- 1.96 standard errors from the mean.

We expect that 90% of the time, our observed mean should be within this range of the target value of 1.0. If our observed mean is more than +/- 1.96 standard errors from the target of 1.0, we can consider these “violations.” An occasional violation is not a cause for concern; however, a series of consecutive violations warrants further analysis.

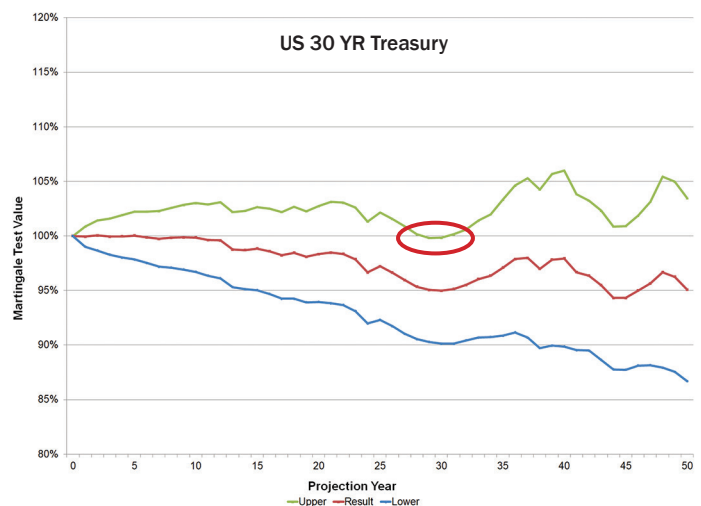
Figure 2 shows an example of a Martingale test with violations.

Figure 1: Martingale Test Result



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Figure 2: Martingale Test Result with Violations



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Looking at the numbers, we see that the upper 5th percentile is less than 1.0 in 2 of the 50 simulated years. The more frequently we test, the more chances there are to observe a violation. Hence, it is the proportion of violations that is relevant, not the absolute number. Since we expect 5% of the results to exceed the upper 5% and only 2/50 or 4.0% exceed this threshold, we can feel comfortable accepting the results.

Risk-Neutral Faster Convergence

Turning on the Risk-Neutral Faster Convergence feature in the GEMS® Economic Scenario Generator (“GEMS®”) for both Treasury and Common Stock models can help a simulation pass the Martingale test by adjusting the random numbers within each of the simulation blocks. It is important to note that using this feature means that the random numbers may not align scenario-by-scenario from one simulation to the next. Thus, for applications where scenario consistency is important, this feature may not be desirable.

Summary

The Martingale test is a key part of validating risk-neutral scenarios. GEMS® can produce a Martingale report which runs Martingale tests on a set of selected asset classes. In addition to displaying the expected value, the report displays the 90% confidence interval to aid in interpreting the results. Using the risk-neutral convergence settings in GEMS® can aid in passing the Martingale test.



Bill Pauling, CFA, is a Director at Conning where he helps clients use Conning’s software tools to make better-informed risk and capital management decisions. Prior to joining Conning in 2021, Mr. Pauling worked at The Hartford where he held a variety of roles related to ERM, ALM and investment risk management. In his most recent role, he was responsible for developing the economic scenarios used in the enterprise economic capital model. Mr. Pauling is a graduate of Drexel University with a BS in Finance. He is a CFA charterholder and a member of the Hartford CFA Society.

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